A NOTE ON sg* CONTINUOUS MAPPINGS IN SOFT TOPOLOGICAL SPACES

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Abstract: The aim of this paper is to introduce sg*closed set in a Soft topological space and to study some of its properties. Then sg* continuous mapping and irresolute mapping areintroduced and some of its properties are studied. The concept *sg open, sg* closed mappings and *sg**homeomorphism are introduced and their properties are studied.**

Key-Words: sg* continuous mapping , irresolute mapping, *sg homeomorphism**

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1 . INTRODUCTION

The theory of soft sets gives a vital mathematical tool for handling uncertainties and vague concepts. In the year 1999, Molodtsov[1] initiated the study of soft sets. Soft set theory has been applied in several directions. Following this Maji, Biswas, and Roy[7,8] discussed soft set theoretical operations and gave an application of soft set theory to a decision making problem. Recently Muhammad Shabir and Munazza Naz introduced the notion of soft topology[10] and established that every soft topology induces a collection of topologies called the parametrized family of topologies induced by the soft topology. Several mathematicians published papers on applications of soft sets and soft topology[1,2,6,11,12,1]. Soft sets and soft topology have applications to data mining, image processing, decision making problems, spatial modeling and neural patterns[3,4,5,7]. In this paper, the concept sg*closed set is introduced in soft topological space and the concept of sg* continuous mapping and *sg** irresolute mapping are introduced and some of their properties are studied. Further the concept *sg** open , sg* closed mappings and *sg**homeomorphism are introduced and some of their basic soft topological properties are investigated. Finally the concept of slightly *sg** continuous mapping is introduced and studied some of its basic concepts.

2. PRELIMINARIES

2.1 Definition A soft set *(A,E)* is called sg* closed in a soft topological space $(X, \tilde{r}\ E)$ of $cl\ (A,E) \tilde{\subseteq} (U,E)$ whenever $(A,E) \tilde{\subseteq} (U,E)$ and (U,E) is soft g open in $\tilde{X}$.

2.2.1 Let $X = \{a_1, a_2, a_3\}, E = \{b_1, b_2\}$ and

$\tilde{r} = \{\tilde{\emptyset}, \tilde{X}, (A_1, E), (A_2, E), (A_3, E), (A_4, E), (A_5, E), (A_6, E), (A_7, E)\}$ where

$(A_1, E) = \{(b_1, \{a_2\}), (b_2, \{a_1\}),$ $\quad$ $(A_2, E) = \{(b_1, \{a_2\}), (b_2, X)\}$

$(A_3, E) = \{(b_1, \{a_{2,3}\}), (b_2, \{a_2, a_3\})\},$ $\quad$ $(A_4, E) = \{(b_1, \{a_1, a_3\}), (b_2, X)\},$

$(A_5, E) = \{(b_1, \emptyset)\{b_2, \{a_1\})$ $\quad$ $(A_6, E) = \{(b_1, \emptyset)\{b_2, \{a_2, a_3\})$ and

$(A_7, E) = \{(b_1, \emptyset), (b_2, X)\}.$

Clearly $(A, E) == \{(b_1, \{a_{1,3}\})(b_2, \{a_3\})\}$ is sg* closed in $(X, \tilde{r}\ E)$.

since for *(A,E)* there exists a soft g open set $(U, E) = \{(b_1, \{a_1, a_3\}. (b_2, \{a_2, a_3\})$ such that $cl\ (A, E) \tilde{\subseteq} (U, E)$.

2.1 Theorem

Every soft closed set is sg* closed in a soft topological space $(X, \tilde{r}\ E)$.

3. sg* CONTINUOUS MAPPINGS

3.1 Definition

A soft mapping $f: \tilde{X} \longrightarrow \tilde{Y}$ is called sg* continuous if $f^1\ (U, E)$ is sg* closed in $(X, \tilde{r}, E)$ for every soft closed set *(U,E)* of $(X, \tilde{\omega}, E)$.

3.2. Theorem

Let $f: \tilde{X} \longrightarrow \tilde{Y}$ be a soft mapping from soft topological space $(X, \tilde{r}, E)$ into a soft topological space $(X, \tilde{r}, E)$. Then the following statements are equivalent.

i) $f: \tilde{X} \longrightarrow \tilde{Y}$ is sg* continuous.

ii) The inverse image of each soft open set in $\tilde{Y}$ is sg* open in $\tilde{Y}$.

iii) For each soft subset $(A, E) \tilde{\in} (Y, \tilde{\omega}, E) sg^* cl(f^{-1}\ (A, E)) \tilde{\subseteq} f^{-1}\ cl(A, E))$.

i v) For each soft subset $(B,E) \tilde{\in}$ $(X, \tilde{r}, E) \mathfrak{f}(sg^{*}cl\ (B,E)) \tilde{\subseteq}\ cl(\mathfrak{f}(B,E))$.

Proof **(i)** $\longrightarrow$ **(ii)** follows from 3.1 Definition.

(i)$\longrightarrow$(iii)

Let *(A,E)* be a soft subset of $(Y, \tilde{\omega}, E)$. By 3.2.1 Definition $\mathfrak{f}^{-1}\ c(A,E))$ is a sg* closed set containing $\mathfrak{f}^{-1}\ (A,E)$ and $sg^{*}cl\ (\mathfrak{f}^{-1}\ (A,E))\ \tilde{\subseteq} \mathfrak{f}^{-1}\ cl(A,E))$.

(iii) $\longrightarrow$(iv)

Let $(B,E) \tilde{\in}\ (Y, \tilde{r}, E)$, then $\mathfrak{f}(B,E) \tilde{\in}\ (Y, \tilde{\omega}, E)$ Hence from (iii) $sg^{*}cl\ (\mathfrak{f}^{-1}\ (\mathfrak{f}(B,E)) \tilde{\subseteq} \mathfrak{f}^{-1}(\ cl(A,E))$. Therefore $\mathfrak{f}(sg^{*}cl(B,E)) \tilde{\subseteq}\ cl\mathfrak{f}\ (B,E)$.

(iv) $\longrightarrow$(i)

Let *(U,E)* be a soft closed set in $\tilde{Y}$. Then by (iv)

$\mathfrak{f}\ (sg^{*}cl\ (\mathfrak{f}^{-1}(U,E))) \tilde{\subseteq}\ cl(\mathfrak{f}(\mathfrak{f}^{-1}(U,E))$. Hence $sg^{*}cl\ (\mathfrak{f}^{-1}(U,E) \tilde{\subseteq}\ \mathfrak{f}^{-1}(U,E)$. Therefore $\mathfrak{f}^{-1}(U,E)$ is a sg* closed set in $\tilde{X}$.

3.3 Theorem

Let $\mathfrak{f}$: $\tilde{X}\ \longrightarrow\ \tilde{Y}$ be a soft continuous mapping from $\tilde{X}$ into $\tilde{Y}$. Then it is sg* continuous.

Proof

(i) $\longrightarrow$ **(ii)** follows from 3.1 Definition.

(i)$\longrightarrow$(iii)

Let *(A,E)* be a soft subset of $(Y, \tilde{\omega}, E)$. By 3.1 Definition $\mathfrak{f}^{-1}(cl\ (A,E)$ is a sg* closed set containing $\mathfrak{f}^{-1}(A,E)$ and $sg^{*}cl\ (\mathfrak{f}^{-1}(A,E)) \tilde{\subseteq}\ \mathfrak{f}^{-,\{1}(cl\ (A,E))$.

(iii) $\longrightarrow$(iv)

Let $(B,E) \tilde{\subseteq}\ (X, \tilde{r}, E)$.Then $\mathfrak{f}(B,E) \tilde{\in}\ (Y, \tilde{\omega}, E)$. Hence from (iii) $sg^{*}cl\ (\mathfrak{f}^{-1}(\mathfrak{f}(B,E)))$ $\tilde{\subseteq}\ \mathfrak{f}^{-1}(cl\mathfrak{f}(B,E))$. Therefore $\mathfrak{f}(sg^{*}cl\ (B,E)) \tilde{\subseteq}\ cl\mathfrak{f}\ (B,E)$.

(iv) $\longrightarrow$(i)

Let *(U,E)* be a soft closed set in $\tilde{Y}$. Then by (iv)

$\mathbf{f}(sg^{*}cl(\mathbf{f}^{-1}\,(U,E))) \mathrel{\tilde{\subseteq}} cl\,(\mathbf{f}\,(\mathbf{f}^{-1}(U,E))$. Hence $g^{*}cl(\mathbf{f}^{-1}\,(U,E) \mathrel{\tilde{\subseteq}} \mathbf{f}(U,E)$.

Therefore $\mathbf{f}^{-1}\,(U,E)$ is a sg* closed set in $\tilde{X}$.

3.4 Theorem

Let $\mathbf{f}: \tilde{X} \longrightarrow \tilde{Y}$ be a soft continuous mapping from $\tilde{X}$ into $\tilde{Y}$. Then it is sg* continuous.

Proof

Let *(A,E)* be any soft closed set in $\tilde{Y}$. Then $\mathbf{f}^{-1}\,(A,E)$ is soft closed in $\tilde{X}$. Therefore by 2.1 Theorem, $\mathbf{f}^{-1}\,(A,E)$ is sg* closed in $\tilde{X}$.

3.5 Example

The following example shows that the converse of the above 3.2.2 Theorem need not be true.

Let $X = \{a_1, a_2, a_3\}, Y = \{a_1, a_2, a_3\}, E = \{b_1, b_2\}$ and

$\tilde{r_1} = \{\tilde{\emptyset}, \tilde{X}, (B_1, E), (B_2, E), (B_3, E), (B_4, E), (B_5, E)\}$

$\tilde{r_1} = \{\tilde{\emptyset}, \tilde{X}, (A_1, E), (A_2, E), (A_3, E), (A_4, E), (A_5, E), (A_6, E)\}$ be two soft topological spaces over X and Y respectively. Then (B_1, E), (B_2, E), (B_3, E), (B_4, E), (B_5, E) are soft sets over X and $(A_1, E), (A_2, E), (A_3, E), (A_4, E), (A_5, E)$ are soft sets over Y defined as follows:

$(A_1, E) = \{(b_1, \{a_2, a_3\}), (b_2, \{a_1, a_3\})\}$, $(A_2, E) = \{(b_1, \{a_3\}), (b_2, \{a_1\})\}$,

$(A_3, E) = \{(b_1, \{a_2\}), (b_2, \{a_3\})\}$, $(A_4, E) = \{(b_1, \{a_3\}), (b_2, \emptyset)\}$,

$(A_5, E) = \{(b_1, X), (b_2, \{a_1, a_3\})\}$, $(A_6, E) = \{(b_1, \{a_{2,3}\}), (b_2, \{a_3\})\}$,

$(B_1, E) = \{(b_1, \{a_2\}), (b_2, \{a_1\})\}$ $(B_2, E) = \{(b_1, \{a_3\}), (b_2, \{a_1, a_3\})\}$,

$(B_3, E) = \{(b_1, \{a_2, a_3\}), (b_2, \{a_1, a_2\})\}$, $(B_4, E) = \{(b_1, X), (b_2, \{a_1, a_2\})\}$,

and $(B_5, E) = \{(b_1, \emptyset), (b_2, \{a_1\})\}$.

Let $\mathbf{f}: \tilde{X} \longrightarrow \tilde{Y}$ be a soft mapping defined by $\mathbf{f}(a_1) = a_1$, $\mathbf{f}(a_2) = a_3$, and $\mathbf{f}(a_3) = a_2$. Then $\mathbf{f}$ is sg* continuous map but not soft continuous. Since $f^{-1}(A_1, E) = \{(b_1, \{a_{2,3}\}), (b_2, \{a_1, a_{2,}\})\}$,

$f^{-1}(A_2, E) = \{(b_1, \{a_2\}), (b_2, \{a_1\})\}$, $f^{-1}(A_3, E) = \{(b_1, \{a_3\}), (b_2, \{a_2\})\}$,

$f^{-1}(A_4, E) = \{(b_1, \{a_2\}), (b_2, \emptyset)\}$, $f^{-1}(A_5, E) = \{(b_1, X), (b_2, \{a_1, a_2\})\}$,

$f^{-1}(A_6, E) = \{(b_1, \{a_2, a_3\}), (b_2, \{a_2\})$ are sg* open sets in $\widetilde{r_1}$ but

$f^{-1}(A_3, E)$, $f^{-1}(A_4, E)$, $f^{-1}(A_5, E)$, $f^{-1}(A_6, E)$ are not soft open sets in $\widetilde{r_1}$.

3.6 Theorem

If $f: \tilde{X} \longrightarrow \tilde{}$ is a sg* continuous mapping from $\tilde{X}$ into $\tilde{Y}$ then f is soft g continuous.

Proof Let (A,E) be any soft closed set in $\tilde{Y}$. Then $f^{-1}(A, E)$ is sg* closed in $\tilde{X}$. Therefore by 2.1 Theorem $f^{-1}(A, E)$is soft g closed in $\tilde{X}$.

3.7 Definition

A soft mapping $f: \tilde{X} \longrightarrow \tilde{}$ called sg* irresolute if $f^{-1}(U, E)$ is sg* closed in $\tilde{X}$ for every sg* closed set of $(Y, \widetilde{\omega}, E)$.

3.8 Remark

A soft mapping $f: \tilde{X} \longrightarrow \tilde{}$ is sg* irresolute if and only if the inverse image of every sg* open set in $(Y, \widetilde{\omega}, E)$ is sg* open in $\tilde{X}$.

3.9 Theorem If $f: \tilde{X} \longrightarrow \tilde{Y}$ and $h: \tilde{Y} \longrightarrow \tilde{Z}$ are any two soft mappings then

i) $h \circ g$ is sg* continuous if h is soft continuous and f is sg* continuous.

ii) $h \circ g$ is sg* continuous if h is sg* continuous and g is sg* irresolute.

iii) $h \circ g$ is sg* irresolute if both g and h are sg* irresolute.

Proof

(i) Let (U,E) be a soft closed set in $\tilde{Z}$. Then $h^{-1}(U, E)$ is soft closed in $\tilde{Y}$ and $g^{-1}(h^{-1}(U, E)) = h^{\circ}\, g)(U, E)$ is sg* closed in $\tilde{X}$.

(ii) Let (U,E) be a soft closed set in $\tilde{Z}$. Then $h^{-1}(U, E)$ is sg* closed in $\tilde{Y}$ and $g^{-1}(h^{-1}(U, E)) = h^{\circ}\, g)(U, E)$ is sg* closed in $\tilde{X}$.

(iii) Let (U,E) be a sg* closed set in $\tilde{Z}$. Then $h^{-1}(U, E)$ is sg* closed in $\tilde{Y}$ and $g^{-1}(h^{-1}(U, E)) = h^{\circ}\, g)(U, E)$ is sg* closed in $\tilde{X}$.

3.10 Theorem

A soft mapping $\boldsymbol{f}: \tilde{X} \longrightarrow \tilde{Y}$ is sg* irresolute if and only if for every soft subset *(U,E)* of $\tilde{X}$, $(sg^* \, cl(U,E)) \tilde{\subseteq} sg^* \, cl(g(U,E))$.

Proof Let g be a sg* irresolute mapping and *(U,E)* be a soft subset in $\tilde{X}$. Then $sg^* \, c(g(U,E))$ is sg* closed set in $\tilde{Y}$. Hence $g^{-1}(sg^* \, cl(g(U,E)))$ is sg* closed in $\tilde{X}$ and $(U,E) \tilde{\subseteq} {}^{-1}(g(U,E)) \tilde{\subseteq} g^{-1}(sg^* \, cl(g(U,E)))$.

Therefore

$sg^* \, cl(U,E) \tilde{\subseteq} g^{-1}(sg^* \, cl(g(U,E)))$, hence $g(sg^* \, cl(U,E)) \tilde{\subseteq} g^{-1}(sg^* \, cl(g(U,E))$.

Conversely, suppose that *(U,E)* is sg* closed in $\tilde{Y}$.

Therefore

$(sg^* \, cl(g^{-1}(U,E))) \tilde{\subseteq} (sg^* \, cl \, (g(g^{-1}(U,E))) = sg^* \, cl(U,E) = (U,E)$. Hence $sg^* \, c(g^{-1}(U,E)) \tilde{\subseteq} g^{-1}(U,E)$.

4. sg* HOMEOMORPHISMS

4.1 Definition

A soft mapping $\boldsymbol{f}: \tilde{X} \longrightarrow \tilde{}$ is called sg* open if $g(U,E)$ of each soft open set *(U,E)* in

$(X, \tilde{r}, E)$ is sg* open in $(Y, \widetilde{\omega}, E)$.

4.2 Definition

A soft mapping $\boldsymbol{f}: \tilde{X} \longrightarrow \tilde{Y}$ is called sg* closed if $g(U,E)$ of each soft closed set *(U,E)* in $(X, \tilde{r}, E)$ is sg* closed in $(Y, \widetilde{\omega}, E)$.

4.3 Theorem

Let the soft mappings $\boldsymbol{f}: \tilde{X} \longrightarrow \tilde{Y}$ and $g: \tilde{Y} \longrightarrow \tilde{Z}$ be bijective. If $g \circ \boldsymbol{f}: \tilde{X} \longrightarrow \tilde{Z}$ is soft continuous and $\boldsymbol{f}: \tilde{X} \longrightarrow \tilde{Y}$ is soft continuous and $\boldsymbol{f}: \tilde{X} \longrightarrow \tilde{Y}$ is sg* closed then $g: \tilde{Y} \longrightarrow \tilde{Z}$ is sg* continuous.

Proof

Let *(U,E)* be the soft closed set in $\tilde{Z}$. Since $g \circ \mathbf{f}: \tilde{X} \longrightarrow \tilde{Z}$ is soft continuous, then $\mathbf{f}^{-1}\,(g^{-1}(U,E)) = (g \circ \mathbf{f})^{-1}(U,E)$ is soft closed set in $\tilde{X}$. Since $\mathbf{f}: \tilde{X} \longrightarrow \tilde{}$ is sg* closed, then $\mathbf{f}\,(\mathbf{f}^{-1}(g^{-1}(U,E))) = g^{-1}\,(U,E)$ is sg* closed set in $\tilde{Y}$.

4.5 Theorem

A soft mapping $\mathbf{f}: \tilde{X} \longrightarrow \tilde{Y}$ is a sg* open iff if $\mathbf{f}(\mathrm{i}kt(B,U)) \tilde{\subseteq} sg^*\mathrm{i}kt\,(\mathbf{f}(B,E))$ for every soft subset *(B,E)* of $\tilde{X}$.

Proof

Let $\mathbf{f}: \tilde{X} \longrightarrow \tilde{Y}$ be sg* open and *(B,E)* be a soft subset of $\tilde{X}$, then $\mathrm{i}kt(B,U)$ is a soft open set in $\tilde{X}$. Hence $\mathbf{f}(\mathrm{i}kt(B,E)) = sg^*\mathrm{i}kt\,(\mathbf{f}(\mathrm{i}kt(B,E))$.

Conversely, Let *(G,E)* be a soft open set in $\tilde{X}$.$\mathbf{f}(G,E) = \mathbf{f}(\mathrm{i}kt(G,E)) \tilde{\subseteq} sg^*\mathrm{i}kt\,(\mathbf{f}(G,E))$, which implies $\mathbf{f}(G,E) \tilde{\subseteq} sg^*\mathrm{i}kt\,(\mathbf{f}(G,E))$. Hence $\mathbf{f}(G,E)$ is a sg* open in $\tilde{Y}$.

4.6 Definition

If a soft mapping $\mathbf{f}: \tilde{X} \longrightarrow \tilde{}$ is sg* continuous bijective and $\mathbf{f}^{-1}$ is sg* continuous then *f* is said to be sg* homeomorphism from $(\mathrm{X}, \tilde{r}, E)$ in to $(Y, \tilde{\varpi}, E)$.

4.7 Theorem

Let $\mathbf{f}: \tilde{X} \longrightarrow \tilde{Y}$ be the soft bijective mapping. Then the following statements are equivalent: . Since f is sg* open map,

i) $\mathbf{f}^{-1}: \tilde{Y} \longrightarrow \tilde{X}$ is sg* continuous.

ii) f is sg* open .

iii) f is sg* closed.

Proof

(i)⟶(ii) Let *(U,E)* be any soft open set in $\tilde{X}$. Since $\mathbf{f}^{-1}: \tilde{Y} \longrightarrow \tilde{X}$ is sg* continuous, therefore $(\mathbf{f}^{-1})^{-1}(U,E) = \mathbf{f}(U,E)$ is sg* open in $\tilde{Y}$.

(ii) ⟶(iii) Let *(B,E)* be any soft closed set in $\tilde{X}$, then $\tilde{X} - (B,E)$ is soft open set in $\tilde{X}$. Since f is sg* open map, $\boldsymbol{f}(\tilde{X} - (B,E))$ is sg* open in $\tilde{Y}$. But $\boldsymbol{f}(\tilde{X} - (B,E)) = \tilde{Y} - \boldsymbol{f}(B,E)$, implies $\boldsymbol{f}(B,E)$is sg* closed in $\tilde{Y}$.

(iii) ⟶(i) Let *(B,E)* be any soft closed set in $\tilde{X}$. Then $(\boldsymbol{f}^{-1})^{-1}(U,E) = \boldsymbol{f}(U,E)$ is sg* closed in $\tilde{Y}$. Therefore $\boldsymbol{f}^{-1}: \tilde{Y} \longrightarrow \tilde{X}$ is sg* continuous.

References

[1] D. Molodtsov, Soft set theory-first results, Math, Appl. 37 (1999),(19-31).
[2] P.K. Maji, R. Biswas and A.R. Roy, Soft set theory, Comput.Math. Appl, 45 (2003),555-562.
[3] M. Shabir and M.Naz, On soft topological spaces. Comput, Math. Appl, 61 (2011) 1786 1799.
[4] A. Aygünoğlu and H.Aygün, Some notes on Soft topological spaces. Neural Comput and Applic., 21 (1) (2012), 113-119
[5] W.K Min, A note on soft topological spaces, Comput. Math.Appl., 62 (2011), 3524-3528.
[6] I,Zorlutuna, M. Akdag, W.K. Min and S. Atmaca, Remarks on soft topological spaces, Ann, Fuzzy Math Inform., 3 (2) (2012), 171 – 185.
[7] S Hussian and B. Ahmad, Some properties of soft topological spaces, Comput. Math. Appl., 62 (2011) 4058 -4067 .
[8] B.Pazar Varol and H. Aygü n, On soft Hausdorff spaces, Ann. Of Fuzzy Math. Inform., 5 (1) (2013), 15 – 24.
[9]B.V.S.T. Sai and V.Srinivasa Kumar, On soft semi-separability, Int.Journalof Math. Analysis, 7 (54) (2013), 2663 – 2669.
[10] N.Levine, Generalized closed sets in topology, Rend. Cric. Mat. Palermo, 19 (2) (1970), 89 -96.
[11] K Kannan, Soft generalized closed sets in soft topological Spaces, Journal of Theoretical And Appl. Inform. Technology, 37 (1) (2012), 17 -21.
[12] S. Alkhazaleh, A. R. Saleh and A.N. Hassan,"Soft multi sets theory", Applied Mathematical Sciences, 5 (72) (2011), 3561 – 3573.