A STUDY ON VERTEX ODD MEAN & VERETX EVEN MEAN LABELING OF SOME GRAPHS

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ABSTRACT

A graph G with p vertices and q edges is a mean graph if there is an injective function f from the vertices of G to $\{0, 1, 2, \dots, q\}$ such that when each edge uv is labeled with $\frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $\frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then the resulting edges are distinct. The graph which admits the vertex odd mean labeling and vertex even mean labeling are called as vertex odd mean and vertex even mean graph respectively. In this paper we investigate vertex odd and vertex even mean labeling of the graphs $P_n + nS_3$, $nC_4 + 2P_2$ and $nC_4 + P_4$.

Keywords: Mean labeling, vertex odd mean labeling, vertex even mean labeling.

1. Introduction

Graphs are one of the prime objects of study in Mathematics. Graph theory is one of the branches of modern mathematics having experienced a most impressive development in recent years. Graph theory serves as a Mathematical Model to represent any system which has a Binary Relation.

A graph is a collection of points and lines connecting some subset of them. The points of a graph are most commonly known as graph vertices, but also be called nodes or simply points. Similarly, the lines connecting the vertices of a graph are most commonly known as graph edges, but may also be called arcs or lines.

The graph structure admits the enormous function that assign the real numbers to the vertices and edges. The process or the practice of assigning integers to the vertices or edges, or both by imposing certain conditions is known as graph labelling. The concept of Graph labeling was first introduced in late 1960s in mathematics world. In the consecutive years dozens of Graph labeling techniques had been studied in more than 800 paper works.

Rosa Alexandra, introduced the techniques of Graph Labeling methods. He defined an evaluation of graph G with q edges as an injection from the vertices of G to the integer set {0, 1, 2, …,, q} such that when each edge of xy is assigned the label |f(x) - f(y)|, and found resulting edge labels are distinct.

Somasundaram and Ponraj have introduced the notion of mean labelings of some graphs. A graph G with p vertices and q edges is called a *mean graph* if there is an injective function f from the vertices of G to $\{0,1, \dots. q\}$ such that when each edge uv is labeled with $\frac{f(u) + f(v)}{2}$ if $f(u) + f(v)$ is even, and $\frac{f(u) + f(v) + 1}{2}$ if $f(u) + f(v)$ is odd, then the resulting edge labels are distinct.[3]

N. Revathi defined a graph $G = (V, E)$ is referred as vertex odd mean and even mean labeling graph with (p, q). A graph G with p vertices and q edges is a mean graph if there is an injective function f from the vertices of G to $\{0, 1, 2, \ldots, q\}$ such that when each edge uv is labeled with $\frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $\frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd then the resulting edges labels are distinct.[7]

Finding out what has been done for any particular kind of labeling and keeping up with new discoveries, in this paper we have investigated Vertex Odd Mean & Vertex Even Mean labeling and extended it for the graphs $P_n + nS_n$, $nC_4 + 2P_2$ and $nC_4 + P_4$.

2. Preliminaries

Definition 2.1:

A **graph** G is an ordered triple set $\{V(G), E(G), X(G)\}$ consisting of non empty set $V(G)$ of vertices, a set $E(G)$ distinct from $V(G)$ of edges and an incidence function X_G that associates with edge of G. If e is an edge and (u, v) are vertices such that $X_G(e) = uv$, then e is said to join the vertices u and v called the ends of e. In this paper we used simple and undirected graphs.

Definition 2.2:

A graph G with p vertices and q edges is a **mean graph** if there is an injective function f from the vertices of G to {0, 1, 2, ..., q} such that when each edge uv is labeled with $\frac{f(u) + f(v)}{2}$ if $f(u) + f(v)$ is even and $\frac{f(u) + f(v) + 1}{2}$ if $f(u)+f(v)$ is odd then the resulting edges are distinct.

Definition 2.3:

A Graph G with q edges to be a **vertex odd mean graph** if there is an injective function f from the vertices of G to $\{1, 3, 5, \ldots, 2q - 1\}$ such that when edge uv is labeled with $\frac{f(u) + f(v)}{2}$ if $f(u) + f(v)$ is even and $\frac{f(u) + f(v) + 1}{2}$ if $f(u) + f(v)$ is odd then the resulting edges are distinct. Such a function is called a vertex odd mean labeling.

Definition 2.4:

A graph G with q edges to be a **vertex even mean graph** if there is an injective function f from the vertices of G to $\{2,4,6,..,2q\}$ such that the induced function $f^*: E(G) \to N$ for each uv assigns a label $f^*(uv) = \frac{f(u)+f(v)}{2}$ results into distinct edge labels. A function this property is called a vertex even mean labeling.

3. Vertex Odd and Vertex Even Mean Labeling of Some Graphs

i) Results on vertex odd mean labeling:

Theorem 3.1: The graph $P_n + nS_3$ is a vertex odd mean graph.

Proof: The graph $P_n + nS_3$ has $4n$ vertices and $4n - 1$ edges.

Let $v_1, v_2, \dots, v_{4n}$ be the vertices of the graph $P_n + nS_3$.

The ordinary labeling of $P_n + nS_3$is given as in figure 1:

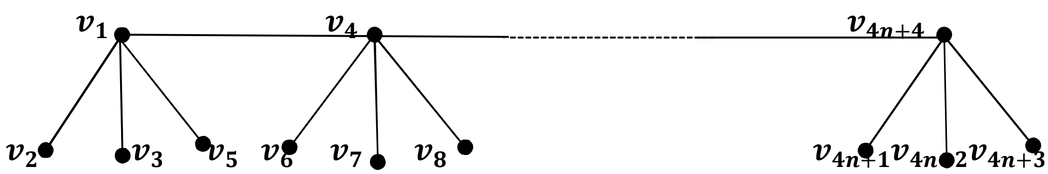

Figure 1: Vertex odd mean labeling of P_n+nS_3

Define a vertex labeling $f: v(P_n + nS_3) \to \{1,3,5, \dots ,2q - 1\}$

By $f(v_i) = 2i - 1, i = 1,2, \dots$4n+3

Then this labeling induces the labelling for the edges according to the definition $f(e_i) = f(uv) = \frac{f(u)+f(v)+1}{2}$. These edge labels are distinct and hence the graph $P_n + nS_3$ is vertex odd mean graph.

Example 3.2: Vertex odd mean labeling of P_2+$2S_3$ is shown in figure 2.

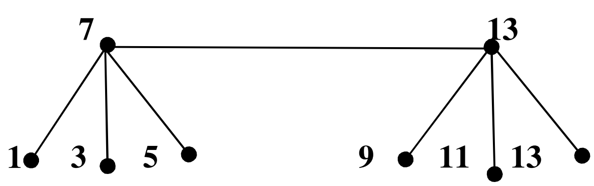

Figure 2: Vertex odd mean labeling of P_2+$2S_3$

Theorem 3.3: The graph $nC_4 + 2P_2$ is a vertex odd mean graph.

Proof: The graph $nC_4 + 2P_2$ has 3n+3 vertices and 4n+2 edges.

Let$v_1, v_2, \dots, v_{3n+3}$ be the vertices of the graph $nC_4 + 2P_2$.

The ordinary labeling of $nC_4 + 2P_2$ is given as in figure 3:

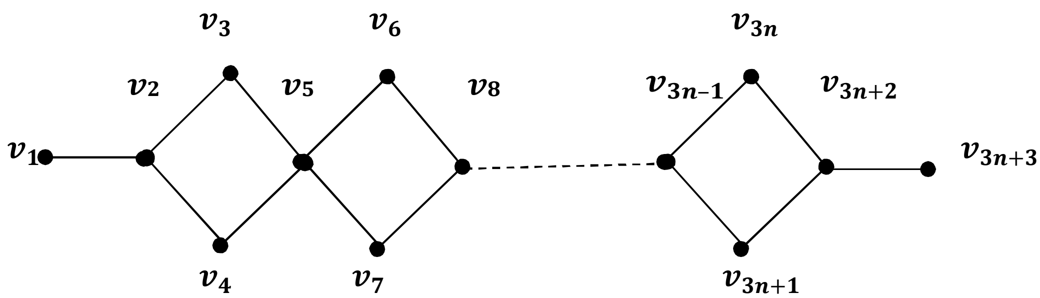

Figure 3: Vertex odd mean labeling of nC_4+2P_2

Define a vertex labeling $f: V(nC_4 + 2P_2) \to \{1,3, \dots ,2q - 1\}$

by $f(v_i) = 2i - 1,\ i = 1,2, \dots \dots 4n+2$

Then this labeling induces edge labeling according to the definition $f(e_i) = f(uv) = \frac{f(u)+f(v)+1}{2}$

Here the graph $nC_4 + 2P_2$ admits vertex odd mean labeling. Hence $nC_4 + 2P_2$ is a vertex odd mean graph.

Example 3.4: Vertex odd mean labeling of $2C_4+2P_2$ is shown in figure 4.

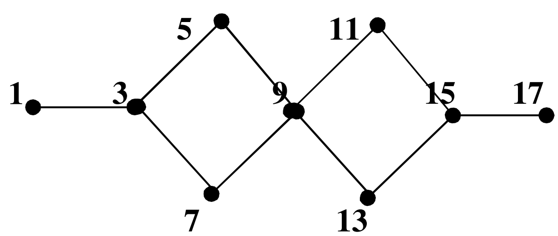

Figure 4: Vertex odd mean labeling of $2C_4+2P_2$

Theorem 4.5: The graph $nC_4 + P_4$ is a vertex odd mean graph.

Proof: the graph $nC_4 + P_4$ has 3n+3 vertices and 4n+3 edges.

Let $v_1, v_2, v_3, \dots v_{3n+3}$ be the vertices of the $nC_4 + P_4$ graph.

The ordinary labeling of $nC_4 + P_4$ is given as in figure 5:

Define a vertex labeling $f: V(nC_4 + P_4) \to \{1,3, \dots ,2q - 1\}$

by $f(v_i) = 2i - 1, i = 1, 2, \dots, 4n+3$

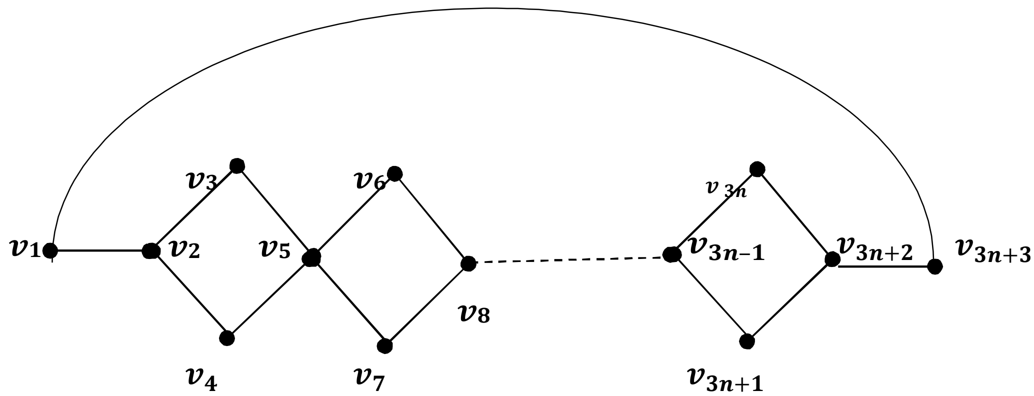

Figure 5: Vertex odd mean labeling of nC4+P4

Then this labeling induces edge labeling according to the definition $f(e_i) = f(uv) = \frac{f(u)+f(v)+1}{2}$

The graph $nC_4 + P_4$ admits vertex odd mean labeling. Hence the graph $nC_4 + P_4$ is vertex odd mean graph.

Example 3.6: Vertex odd mean labeling of $2C_4+P_4$ is shown in figure 6.

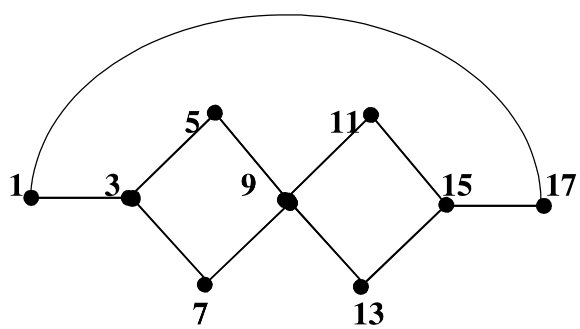

Figure 6: Vertex odd mean labeling of 2C4+P4

ii) Results on vertex even mean labeling:

Theorem 3.7: The graph $P_n + nS_3$ is a vertex even mean graph.

Proof: The graph $P_n + nS_3$ has $4n$ vertices and $4n - 1$ edges.

Let $v_1, v_2, \dots, v_{4n}$ be the vertices and $e_1, e_2, \dots, e_{4n-1}$ edges of the graph $P_n + nS_3$.

Define a vertex labeling $f: v(P_n + nS_3) \to \{2, 4, 6, \dots, 2q\}$

by $f(v_i) = 2i, \quad i = 1, 2, \dots 7, 9, \dots, 4n - 1$ and for $i \neq 8, 12, 16, \dots, 4n$

The ordinary labeling of $P_n + nS_3$is given as in figure 7:

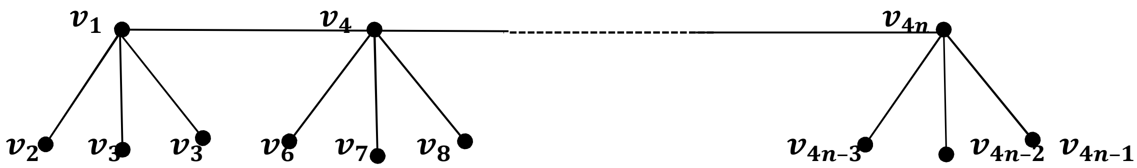

Figure 7: Vertex even mean labeling of P_n+nS_3

This vertex labeling induces edge labeling according to the definition$f(e_i) = f(uv) = \frac{f(u)+f(v)}{2}$.

Here the graph $P_n + nS_3$ admits vertex even mean labeling. Hence $P_n + nS_3$ is vertex even mean graph.

Example 3.8: Vertex even mean labeling of the graph $P_2 + 2S_3$ is shown in figure 8.

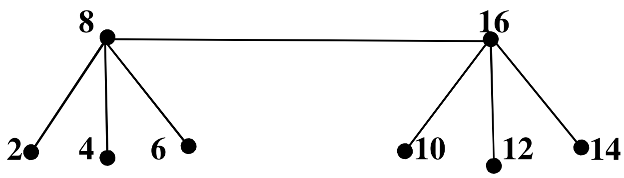

Figure 8: Vertex even mean labeling of P_2+$2S_3$

Theorem 3.9: The graph $nC_4 + 2P_2$ is a vertex even mean graph.

Proof: The graph $nC_4 + 2P_2$ has $6n + 6$ vertices and 6n + 5 edges.

Let $v_1, v_2, \dots, v_{6n+6}$ be the vertices of the graph $nC_4 + 2P_2$.

The ordinary labeling of $nC_4 + 2P_2$ is given as in figure 9:

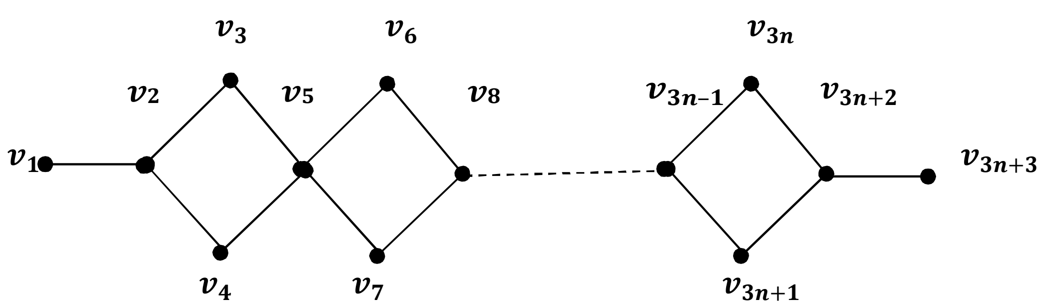

Figure 9: Vertex even mean labeling of nC_4+2P_2

Define a vertex labeling $f: V(nC_4 + 2P_2) \rightarrow \{2, 4, \dots, 2q\}$

by $f(v_i) = 2i,\ i = 1, 2, \dots, 4n+2$.

Then this labeling induces edge labeling according to the definition

$f(e_i) = f(uv) = \frac{f(u)+f(v)}{2}$.

The graph $nC_4 + 2P_2$ admits vertex even mean labeling. Hence $nC_4 + 2P_2$ is vertex even mean graph.

Example 3.10: Vertex even mean labeling of $2C_4 + 2P_2$ is shown in figure 10.

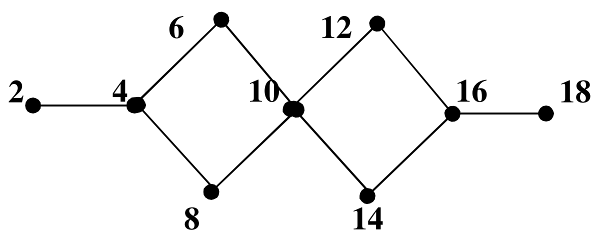

Figure 10: Vertex even mean labeling of $2C_4+2P_2$

Theorem 3.11: The graph $nC_4 + P_4$ is a vertex even mean graph.

Proof: The graph $nC_4 + P_4$ has 3n+3 vertices and 4n+3 edges.

Let $v_1, v_2, v_3, \dots v_{3n+3}$ be the vertices of the graph $nC_4 + P_4$.

The ordinary labeling of $nC_4 + P_4$ is given as in figure 11:

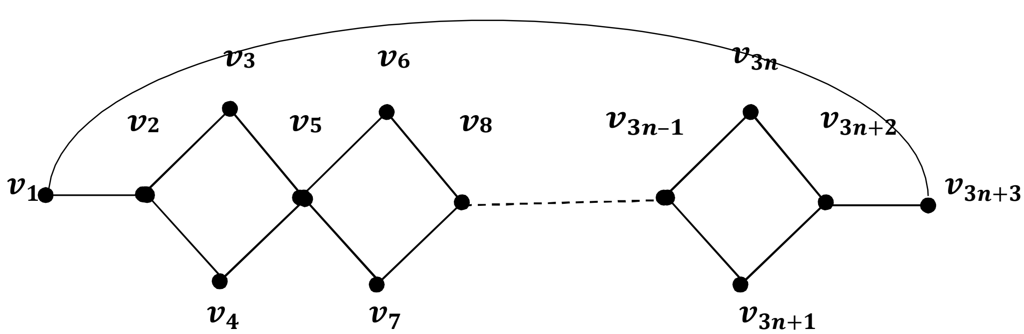

Figure 11: Vertex even mean labeling of nC_4+P_4

Define a vertex labeling $f: V(nC_4 + P_4) \rightarrow \{2, 4, \dots, 2q\}$

by $f(v_i) = 2i,\ i = 1, 2, \dots, 4n + 3$

Then this labeling induces edge labeling according to the definition $f(e_i) = f(uv) = \frac{f(u)+f(v)}{2}$.

Here the graph $nC_4 + P_4$ admits vertex even mean labeling. Hence the graph $nC_4 + P_4$ is vertex even mean graph.

Example 3.12: Vertex even mean labeling of $2C_4 + P_4$ is shown in figure 12.

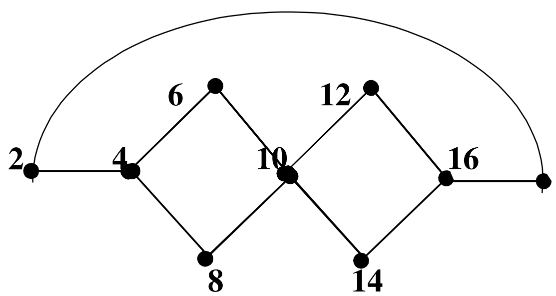

Figure 12: Vertex even mean labeling of 2C4+P4

CONCLUSION:

In this paper we have proved the graphs $P_n + nS_n$, $nC_4 + 2P_2$ and $nC_4 + P_4$ are vertex odd mean and vertex even mean graphs.

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